Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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Pearson

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1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\frac{2}{\sqrt{9-2x}} \qquad |x| < \frac{9}{2}$$

giving each coefficient as a simplified fraction.

(5)

By substituting $x = 1$ into the answer to part (a),

(b) find an approximation for $\sqrt{7}$, giving your answer to 4 decimal places.

(2)

(a) Get the formula for the Binomial Expansion from the formula booklet:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$, Range of validity $|x|<1$

We see that the given expansion is in form $(a+bx)^n$, we need to manipulate to get $(1+ax)^n$.

$$2(9-2x)^{-\frac{1}{2}} = 2(9)^{-\frac{1}{2}}(1-\frac{2}{9}x)^{-\frac{1}{2}}$$

$$= \frac{2}{3}(1-\frac{2}{9}x)^{-\frac{1}{2}} \quad \text{B1}$$

Substitute into the Formula:

$$\frac{2}{3}(1-\frac{2}{9}x)^{-\frac{1}{2}} = \frac{2}{3}\left(1+(-\tfrac{1}{2})(-\tfrac{2}{9}x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\tfrac{2}{9}x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-\tfrac{2}{9}x)^3 + \ldots\right) \quad \text{M1}$$

$$= \frac{2}{3}\left(1 + \frac{1}{9}x + (\frac{3}{8})(\frac{4}{81}x^2) + (-\frac{5}{16})(-\frac{8}{729}x^3) + \ldots\right)$$

$$= \frac{2}{3}\left(1 + \frac{1}{9}x + \frac{1}{54}x^2 + \frac{5}{1458}x^3 + \ldots\right) \quad \text{A1}$$

$$= \frac{2}{3} + \frac{2}{27}x + \frac{1}{81}x^2 + \frac{5}{2187}x^3 + \ldots \quad \text{A1A1}$$

(b) Substitute into LHS:

$$\frac{2}{\sqrt{9-2(1)}} = \frac{2}{\sqrt{7}} \Rightarrow \sqrt{7} = 2\cdot\left(\frac{2}{\sqrt{7}}\right)^{-1}$$

Hence we will need $\sqrt{7} = 2\cdot(\text{our expansion})^{-1}$ to get the required approximation.

Substitute $x=1$ into our expansion:

$$\frac{2}{\sqrt{9-2(1)}} = \frac{2}{3} + \frac{2}{27}(1) + \frac{1}{81}(1)^2 + \frac{5}{2187}(1)^3 \quad \text{M1}$$

$$= \frac{1652}{2187}$$

$$\sqrt{7} = 2 \times \left(\frac{1652}{2187}\right)^{-1}$$

$= 2.6477$ to 4dp. A1

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Question 1 continued

(Total 7 marks)

Q1

2. The curve C has parametric equations

$$x = \frac{t^4}{2t+1} \qquad y = \frac{t^3}{2t+1} \qquad t > 0$$

(a) Write down $\frac{x}{y}$ in terms of t, giving your answer in simplest form.

(1)

(b) Hence show that all points on C satisfy the equation

$$x^3 - 2xy^3 - y^4 = 0$$

(3)

(a)

$$\frac{x}{y} = \frac{\frac{t^4}{\cancel{2t+1}}}{\frac{t^3}{\cancel{2t+1}}}$$

$$= \frac{t^4}{t^3}$$

$$= t \quad \text{B1}$$

(b) We got that $t = \frac{x}{y}$. Use this to get y in terms of x and y:

$$y = \frac{\left(\frac{x}{y}\right)^3}{2\left(\frac{x}{y}\right)+1} = \frac{\frac{x^3}{y^3}}{\frac{2x}{y}+\frac{y}{y}} \quad \text{M1}$$

$$= \frac{\frac{x^3}{y^{\cancel{3}\,2}}}{\frac{2x+y}{\cancel{y}}} = \frac{x^3}{y^2(2x+y)}$$

$$\Rightarrow y = \frac{x^3}{2xy^2+y^3}$$ y in terms of x and y.

M1

Now Rearrange:

$$y(2xy^2+y^3) = x^3$$

$$2xy^3+y^4 = x^3$$

$$\Rightarrow x^3-2xy^3-y^4=0 \quad \text{A1}$$

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Question 2 continued

Q2

(Total 4 marks)

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3. The curve C has equation

$$3y^2 - 11x^2 + 11xy = 20y - 36x + 28$$

(a) Find, in simplest form, $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **(5)**

The point $P(4, k)$, where k is a constant, lies on C.

Given that $k < 0$

(b) find the value of the gradient of C at P **(5)**

(a) ☆ Implicit differentiation:

When differentiating y, multiply by $\frac{dy}{dx}$

Multiplication ∴ Product Rule.

$$3y^2 - 11x^2 + 11xy = 20y - 36x + 28 \qquad \frac{d}{dx}(uv) = uv' + u'v$$

$$6y\frac{dy}{dx} - 22x + 11y + 11x\frac{dy}{dx} = 20\frac{dy}{dx} - 36 \qquad \text{M1M1A1}$$

$$6y\frac{dy}{dx} + 11x\frac{dy}{dx} - 20\frac{dy}{dx} = 22x - 11y - 36$$ Rearrange to get $\frac{dy}{dx}$ on one side

$$\frac{dy}{dx}(6y + 11x - 20) = 22x - 11y - 36$$ Factor $\frac{dy}{dx}$ out

M1

$$\frac{dy}{dx} = \frac{22x - 11y - 36}{6y + 11x - 20} \qquad \text{A1}$$

(b) To get k substitute x=4, y=k into the given equation:

$$3k^2 - 11(4)^2 + 11(4)(k) = 20k - 36(4) + 28 \qquad \text{M1}$$

$$3k^2 - 176 + 44k - 20k + 144 - 28 = 0$$

$$\Rightarrow 3k^2 + 24k - 60 = 0$$ Quadratic

÷3

$$k^2 + 8k - 20 = 0 \qquad \text{M1}$$

$$(k + 10)(k - 2) = 0$$

$$k = -10 \qquad k = 2 \qquad \text{A1}$$

Since we're given that $k<0$, $\underline{k = -10}$.

Hence $P(4,-10)$. Substitute into $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{22(4) - 11(-10) - 36}{6(-10) + 11(4) - 20}$$

$$= \frac{88 + 110 - 36}{-60 + 44 - 20} \qquad \text{M1}$$

$$\text{A1} \qquad \frac{dy}{dx} = -\frac{9}{2}$$ Gradient of C at P.

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Question 3 continued

Q3

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4. $$\mathrm{f}(x) = \frac{4 - 4x}{x(x-2)^2} \qquad x > 2$$

(a) Express f(x) in partial fractions.
(4)

(b) Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$
(3)

(c) Find

$$\int_3^5 \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer in the form $a + \ln b$, where a and b are rational numbers to be found.
(2)

(a) We are asked to do Partial Fractions

$$\frac{4-4x}{x(x-2)^2} = \underbrace{\frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}}$$ B1

We realize this is a repeated root, so leave one root squared, the other unsquared

$$\frac{4-4x}{x(x-2)^2} = \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2}$$

Equate the numerators and substitute in values in order to get A, B and C.

$$4-4x = A(x-2)^2 + Bx(x-2) + Cx$$

choose values to substitute in so that brackets become 0, to make your life easier!

$x=2 \Rightarrow 4-4(2) = A(\cancel{2-2})^2 + B(2)(\cancel{2-2}) + C(2)$

$-4 = 2C \Rightarrow C = -2$ M1

$x=0 \Rightarrow 4-4(0) = A(0-2)^2 + \cancel{B(0)(0-2)} + \cancel{C(0)}$

$4 = 4A \Rightarrow A = 1$ A1

$x=1 \Rightarrow 4-4(1) = 1(1-2)^2 + B(1)(1-2) - 2(1)$

$0 = 1 - B - 2 \Rightarrow B = -1$

$$\Rightarrow \frac{4-4x}{x(x-2)^2} = \frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2}$$ A1

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Question 4 continued

(b) Use the answer to part (a) and integrate the partial fractions.

$$\int \frac{4-4x}{x(x-2)^2}\,dx = \int \frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2}\,dx$$

$= -2(x-2)^{-2}$

$$= \ln x - \ln(x-2) - \frac{2}{-1}(x-2)^{-1}$$

$$= \ln x - \ln(x-2) + \frac{2}{x-2} + c$$

M1 M1 A1

(c)

$$\int_3^5 \frac{4-4x}{x(x-2)^2}\,dx$$

$$= \left[\ln x - \ln(x-2) + \frac{2}{x-2}\right]_3^5$$

we already integrated in (b)

$$= \left(\ln 5 - \ln(5-2) + \frac{2}{5-2}\right) - \left(\ln 3 - \ln(3-2) + \frac{2}{3-2}\right)$$

$$= \ln 5 - \ln 3 + \frac{2}{3} - \ln 3 + \cancel{\ln 1}^{\,0} - 2$$

M1

$$= \ln 5 - 2\ln 3 - \frac{4}{3}$$

$$= \ln 5 - \ln 3^2 - \frac{4}{3}$$

apply ln law $a\ln b = \ln b^a$

A1

$$= \ln \frac{5}{9} - \frac{4}{3}$$

apply ln law $\ln a - \ln b = \ln \frac{a}{b}$

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Question 4 continued

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Question 4 continued

Q4

(Total 9 marks)

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5. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} 13 \\ -1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A. **(6)**

(b) Find the acute angle between l_1 and l_2, giving your answer in degrees to one decimal place. **(3)**

A circle with centre A and radius 35 cuts the line l_1 at the points P and Q.

Given that the x coordinate of P is greater than the x coordinate of Q,

(c) find the coordinates of P and the coordinates of Q. **(4)**

(a) Equate the general points of the line:

$$\underbrace{\begin{pmatrix} 4+2\lambda \\ 4-3\lambda \\ -5+6\lambda \end{pmatrix}}_{l_1} = \underbrace{\begin{pmatrix} 13+5\mu \\ -1+\mu \\ 4-3\mu \end{pmatrix}}_{l_2}$$

Simultaneously solve x and y component equations to get μ and λ values:

$4+2\lambda = 13+5\mu \quad |\cdot 3| \quad 12+\cancel{6\lambda} = 39+15\mu$ use elimination

$4-3\lambda = -1+\mu \quad |\cdot 2| \quad 8-\cancel{6\lambda} = -2+2\mu$ + M1M1

$20 = 37+17\mu$

$-17 = 17\mu \Rightarrow \mu = -1$

$8-6\lambda = -2+2(-1)$

$8-6\lambda = -4$

$6\lambda = 12 \Rightarrow \lambda = 2$ A1

To prove that they intersect substitute μ and λ into the equation for z:

$-5+6(2) = 4-3(-1)$

$7 = 7 \quad \therefore$ lines l_1 and l_2 intersect B1

To get value of A use l_1:

$\vec{OA} = \begin{pmatrix} 4 \\ 4 \\ -5 \end{pmatrix} + (2)\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ M1

$\vec{OA} = \begin{pmatrix} 8 \\ -2 \\ 7 \end{pmatrix}$ position vector of point A

A1

☆ If you are unsure here you can also check with l_2 and your μ to make sure.

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Question 5 continued

(b) Use Formula:

$\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}$ Formula for dot product: $\begin{pmatrix}a\\b\\c\end{pmatrix}\cdot\begin{pmatrix}d\\e\\f\end{pmatrix} = ad+be+cf$

Formula for Magnitude of a vector:

Let $\vec{AB} = \begin{pmatrix}a\\b\\c\end{pmatrix}$.

$|\vec{AB}| = \sqrt{a^2+b^2+c^2}$

We will use the two direction vectors:

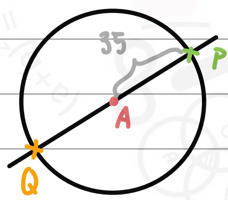
$$\cos\theta = \left|\frac{\begin{pmatrix}2\\-3\\6\end{pmatrix}\cdot\begin{pmatrix}5\\1\\-3\end{pmatrix}}{\sqrt{2^2+3^2+6^2}\sqrt{5^2+1^2+3^2}}\right| \quad \text{M1}$$

$$= \left|\frac{10-3-18}{\sqrt{49}\sqrt{35}}\right|$$

$$\theta = \cos^{-1}\left|\frac{-11}{7\sqrt{35}}\right| \quad \text{A1}$$

$$\theta = 74.6^\circ \quad \text{A1}$$

(c) Diagram



The length AP/AQ is a multiple of the magnitude of the direction vector of the line l_1.

Get the magnitude of the direction vector:

$\sqrt{2^2+3^2+6^2} = \sqrt{49} = 7$. M1

The radius is given to be 35, hence 5 times the direction vector, $\therefore \lambda = 5$ at P, and -5 at Q. M1

Hence:

$\vec{OP} = \begin{pmatrix}4\\4\\-5\end{pmatrix} + (5)\begin{pmatrix}2\\-3\\6\end{pmatrix} = \begin{pmatrix}18\\-17\\37\end{pmatrix}$

$\vec{OQ} = \begin{pmatrix}4\\4\\-5\end{pmatrix} + (-5)\begin{pmatrix}2\\-3\\6\end{pmatrix} = \begin{pmatrix}-2\\13\\-23\end{pmatrix}$

$\therefore$ Coordinates: $P(18,-17,37)$ A1

$Q(-2,13,-23)$ A1

Turn over

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Question 5 continued

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Question 5 continued

Q5

(Total 13 marks)

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6. Use integration by parts to show that

$$\int e^{2x}\cos 3x \,dx = pe^{2x}\sin 3x + qe^{2x}\cos 3x + k$$

where p and q are rational numbers to be found and k is an arbitrary constant.

(6)

$$\int e^{2x}\cos 3x \,dx$$

★ Integration by Parts:
$\int vu'\,dx = vu - \int uv'\,dx$

$u = e^{2x}$ $u' = 2e^{2x}$

$v = \frac{1}{3}\sin 3x$ $v' = \cos 3x$

$$= \frac{e^{2x}}{3}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\,dx$$

By Parts again

M1A1

$u = e^{2x}$ $u' = 2e^{2x}$

$v = -\frac{1}{3}\cos 3x$ $v' = \sin 3x$

$$= \frac{e^{2x}}{3}\sin 3x - \frac{2}{3}\left(-\frac{e^{2x}}{3}\cos 3x - \int -\frac{2e^{2x}}{3}\cos 3x\,dx\right)$$ M1

$$= \frac{e^{2x}}{3}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{2}{3}\cdot\frac{2}{3}\int e^{2x}\cos 3x\,dx$$

This is our initial integral.

Reintroduce our initial Integral on the LHS and use it to get rid of the infinite "By Parts".

$$\int e^{2x}\cos 3x\,dx = \frac{e^{2x}}{3}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}\int e^{2x}\cos 3x\,dx$$ A1

$$\int e^{2x}\cos 3x\,dx + \frac{4}{9}\int e^{2x}\cos 3x\,dx = \frac{e^{2x}}{3}\sin 3x + \frac{2}{9}e^{2x}\cos 3x$$

$$\frac{13}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x$$ M1

$$\int e^{2x}\cos 3x\,dx = \frac{9}{13}\left(\frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x\right)$$

$$\int e^{2x}\cos 3x\,dx = \frac{3}{13}e^{2x}\sin 3x + \frac{2}{13}e^{2x}\cos 3x + k$$

A1 hence shown

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Question 6 continued

Q6

(Total 6 marks)

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7. Water is flowing into a large container and is leaking from a hole at the base of the container.

At time t seconds after the water starts to flow, the volume, V cm^3, of water in the container is modelled by the differential equation

$$\frac{dV}{dt} = 300 - kV$$

where k is a constant.

(a) Solve the differential equation to show that, according to the model,

$$V = \frac{300}{k} + Ae^{-kt}$$

where A is a constant.

(5)

Given that the container is initially empty and that when $t = 10$, the volume of water is increasing at a rate of $200\,\text{cm}^3\,\text{s}^{-1}$

(b) find the exact value of k.

(4)

(c) Hence find, according to the model, the time taken for the volume of water in the container to reach 6 litres. Give your answer to the nearest second.

(2)

(a) $\frac{dV}{dt} = 300 - kV$

$\int \frac{1}{300-kV}\,dV = \int 1\,dt$ Separate variables by grouping the like terms on one side

B1

Reverse chain rule: Divide by the derivative of the bracket. $-\frac{1}{k}\ln(300-kV) = t + c$ Integrate both sides

M1A1

$\ln(300-kV) = -kt - kc$ Multiply both sides by $-k$

$e^{\ln(300-kV)} = e^{-kt+d}$ where $d = -kc$

M1 $300 - kV = e^{-kt+d}$ use ln law: $e^{\ln x} = x$

$kV = 300 - e^{-kt+d}$ Rearrange

$V = \frac{300}{k} - \frac{1}{k}e^{-kt+d}$

$V = \frac{300}{k} - \frac{1}{k}\left(e^{-kt}\cdot e^{d}\right)$ Use index law $e^{a+b} = e^a \cdot e^b$

$\therefore V = \frac{300}{k} + Ae^{-kt}$ where $A = -\frac{e^d}{k}$

A1 hence shown

P 7 0 3 4 9 A 0 1 8 2 8

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Question 7 continued

(b) Given that at $t=0$, $V=0$ and at $t=10$, $\frac{dV}{dt}=200$

First use $t=0, V=0$ with our solved D. E.:

$$0 = \frac{300}{k} + Ae^{-k(0)}$$

1

$$\Rightarrow A = -\frac{300}{k}$$ M1

Now use $t=10$, $\frac{dV}{dt}=200$ with the given $\frac{dV}{dt}$ ...

$$200 = 300 - kV$$

$$\Rightarrow kV = 100$$ M1

Now substitute $t=10$, $A = -\frac{300}{k}$ and $kV = 100$ into our solved D.E. and solve for k:

$$V = \frac{300}{k} + Ae^{-kt}$$

×k ×k Multiply both sides by k

$$kV = 300 + Ake^{-kt}$$

$$\Rightarrow 100 = 300 - \frac{300}{\cancel{k}} \cdot \cancel{k}e^{-k(10)}$$ M1

$$100 = 300 - 300e^{-10k}$$

$$300e^{-10k} = 200$$

$$e^{-10k} = \frac{2}{3}$$

$$\ln e^{-10k} = \ln\frac{2}{3}$$ use $\ln e^{x} = x$

$$-10k = \ln\frac{2}{3}$$

$$\therefore k = -\frac{1}{10}\ln\frac{2}{3}$$ A1

P 7 0 3 4 9 A 0 1 9 2 8

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Question 7 continued

(c) Use our solved D.E. with the value of k we found.

$$k = -\frac{1}{10}\ln\frac{2}{3} \Rightarrow k = \frac{1}{10}\ln\frac{3}{2}$$ Use ln law: $a\ln b = \ln b^{a}$

$$V = \frac{300}{\frac{1}{10}\ln\frac{3}{2}} - \frac{300}{\frac{1}{10}\ln\frac{3}{2}} e^{(-\frac{1}{10}\ln\frac{3}{2})t}$$

Substitute V = 6000 and solve for t:

$$6000 = \frac{3000}{\ln\frac{3}{2}} - \frac{3000}{\ln\frac{3}{2}} e^{-\frac{t}{10}\ln\frac{3}{2}}$$ M1

$$6000 = \frac{3000}{\ln\frac{3}{2}}\left(1 - e^{-\frac{t}{10}\ln\frac{3}{2}}\right)$$

$$\frac{\overset{2}{\cancel{6000}} \cdot \ln\frac{3}{2}}{\cancel{3000}} = 1 - e^{-\frac{t}{10}\ln\frac{3}{2}}$$

$$e^{-\frac{t}{10}\ln\frac{3}{2}} = 1 - 2\ln\frac{3}{2}$$

$$\ln e^{-\frac{t}{10}\ln\frac{3}{2}} = \ln(1 - 2\ln\tfrac{3}{2})$$

$$\Rightarrow -\frac{t}{10}\ln\frac{3}{2} = \ln(1 - 2\ln\tfrac{3}{2})$$

$$t = \frac{-10\ln(1 - 2\ln\frac{3}{2})}{\ln\frac{3}{2}}$$

$$\boxed{\Rightarrow t = 41\text{ s}}$$ A1

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Question 7 continued

Q7

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8. Use proof by contradiction to prove that, for all positive real numbers x and y,

$$\frac{9x}{y} + \frac{y}{x} \geqslant 6$$

(4)

Make an Assumption:

"Assume that there exist positive real numbers x and y that satisfy $\frac{9x}{y} + \frac{y}{x} < 6$" B1

We want our assumption to be the opposite of what we are trying to prove. In the Proof, we are trying to contradict the assumption

Let's solve the inequality:

$$\frac{9x}{y} + \frac{y}{x} < 6$$

$\cdot xy$

No need to flip the sign as x and y are positive

M1 $9x^2 + y^2 < 6xy$

$9x^2 - 6xy + y^2 < 0$

A1 $(3x - y)^2 < 0$ Factorize

This is a contradiction, since any real number squared is positive, and x and y are real.

Hence, $\frac{9x}{y} + \frac{y}{x} < 0$ must be false and $\frac{9x}{y} + \frac{y}{x} \geqslant 6$ is proven to be true by contradiction.

A1

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Question 8 continued

(Total 4 marks)

Q8

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9.

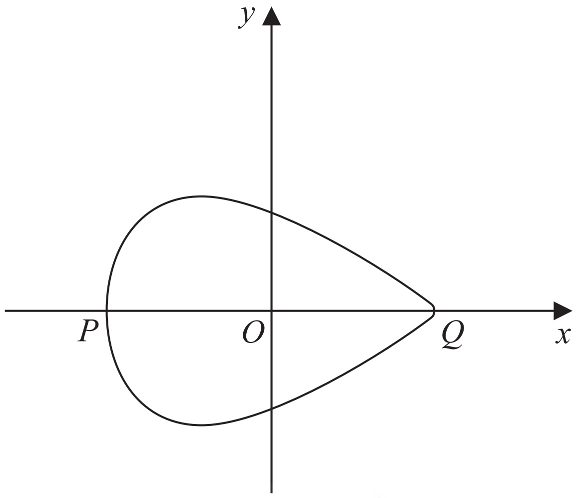

Figure 1

Figure 1 shows a sketch of a closed curve with parametric equations

$$x = 5\cos\theta \qquad y = 3\sin\theta - \sin 2\theta \qquad 0 \leqslant \theta < 2\pi$$

The region enclosed by the curve is rotated through π radians about the x-axis to form a solid of revolution.

(a) Show that the volume, V, of the solid of revolution is given by

$$V = 5\pi \int_{\alpha}^{\beta} \sin^3\theta\,(3 - 2\cos\theta)^2 \, \mathrm{d}\theta$$

where α and β are constants to be found.

(4)

(b) Use the substitution $u = \cos\theta$ and algebraic integration to show that $V = k\pi$ where k is a rational number to be found.

(7)

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Question 9 continued

(a) Formula for Volume of Revolution around the x-axis:

$$V = \pi \int_a^b y^2 \, dx$$

★ Parametric Integration

$$\text{area} = \int_a^b y \frac{dx}{dt} dt$$

where a and b are the points on the x-axis bounding the area

Let's get $\frac{dx}{d\theta}$:

$$x = 5\cos\theta$$

$$\frac{dx}{d\theta} = -5\sin\theta$$

Let's get limits. On the x-axis, hence when $y = 0$.

$$0 = 3\sin\theta - \sin 2\theta$$

$$0 = 3\sin\theta - 2\sin\theta\cos\theta$$ Use double angle formula: $\sin 2A = 2\sin A\cos A$

$$0 = \sin\theta(3 - 2\cos\theta)$$

$$\sin\theta = 0 \Rightarrow \theta = 0, \pi.$$ → Check which limit is which:

$x = 5\cos(\pi) = -5$ Lower

$x = 5\cos(0) = 5$ upper

Hence use the formula:

$$V = \pi \int_\pi^0 (3\sin\theta - \sin 2\theta)^2 (-5\sin\theta) \, d\theta$$ M1

$$= -5\pi \int_\pi^0 (3\sin\theta - 2\sin\theta\cos\theta)^2 \cdot \sin\theta \, d\theta$$ expand bracket, take out the -5. M1A1

$$= -5\pi \int_\pi^0 \sin^2\theta(3 - 2\cos\theta)^2 \cdot \sin\theta \, d\theta$$ factor out $\sin^2\theta$

$$= -1\left(-5\pi \int_\pi^0 \sin^3\theta(3 - 2\cos\theta)^2 d\theta\right)$$ flip the limits by multiplying the integral by -1.

$$= 5\pi \int_0^\pi \sin^3\theta(3 - 2\cos\theta)^2 d\theta$$ hence shown

A1

Leave blank

Question 9 continued

(b)

To use the given substitution $u = \cos\theta$, we need to find new limits and $d\theta = \ldots$

New Limits:

θ		u
0	$\cos(0) \rightarrow$	1
π	$\cos(\pi) \rightarrow$	-1

M1

To get dx, differentiate $\cos\theta$:

$$u = \cos\theta$$

$$\frac{du}{d\theta} = -\sin\theta \Rightarrow d\theta = \frac{du}{-\sin\theta}$$

Substitute into the Integration:

$$5\pi \int_0^{\pi} \sin^3\theta (3 - 2\cos\theta)^2 d\theta$$

limits, $u = \ldots$, $d\theta$

$$= 5\pi \int_1^{-1} \sin^3\theta (3 - 2u)^2 \cdot \frac{du}{-\sin\theta}$$ M1

$$= -5\pi \int_1^{-1} \sin^2\theta (3 - 2u)^2 du$$ $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$u = \ldots$

$$= 5\pi \int_{-1}^{1} (1 - \cos^2\theta)(3 - 2u)^2 du$$ flip limits by multiplying by -1

$$= 5\pi \int_{-1}^{1} (1 - u^2)(9 - 12u + 4u^2) du$$ expand

A1

$$= 5\pi \int_{-1}^{1} 9 - 12u + 4u^2 - 9u^2 + 12u^3 - 4u^4 \, du$$ expand some more

M1A1

$$= 5\pi \int_{-1}^{1} 9 - 12u - 5u^2 + 12u^3 - 4u^4 \, du$$ collect like terms

M1 $$= 5\pi \left[9u - 6u^2 - \frac{5}{3}u^3 + 3u^4 - \frac{4}{5}u^5 \right]_{-1}^{1}$$

★ Simple Integration: $\int x^n \, dx = \frac{1}{n} x^{n+1} + c$

$$= 5\pi \left(\left(9(1) - 6(1)^2 - \frac{5}{3}(1)^3 + 3(1)^4 - \frac{4}{5}(1)^5\right) - \left(9(-1) - 6(-1)^2 - \frac{5}{3}(-1)^3 + 3(-1)^4 - \frac{4}{5}(-1)^5\right) \right)$$

$$= 5\pi \left(9 - \cancel{6} - \frac{5}{3} + \cancel{3} - \frac{4}{5} + 9 + \cancel{6} - \frac{5}{3} - \cancel{3} - \frac{4}{5} \right)$$

$$= 5\pi \left(18 - \frac{10}{3} - \frac{8}{5} \right)$$

$$\boxed{V = \frac{196\pi}{3}}$$ found.

A1

P 7 0 3 4 9 A 0 2 6 2 8

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Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END